K18

Q1 • $\cos\left(\frac{5\pi}{3}\right) = \cos\left(-\frac{\pi}{3}\right) = \frac{1}{2}$

• $\sin\left(\frac{-3\pi}{4}\right) = -\sin\left(\frac{3\pi}{4}\right) = -\sin\left(\pi - \frac{\pi}{4}\right) = -\sin\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$

• $\sin\left(\frac{17\pi}{6}\right) = \sin\left(\frac{5\pi}{6} + 2\pi\right) = \sin\left(\pi - \frac{\pi}{6}\right) = \sin\frac{\pi}{6} = \frac{1}{2}$

• $\cos\left(a + \frac{\pi}{2}\right) = -\sin a$

• $\tan\left(a + \frac{\pi}{2}\right) = \frac{\sin\left(a+\frac{\pi}{2}\right)}{\cos\left(a+\frac{\pi}{2}\right)} = \frac{\cos a}{-\sin a} = -\frac{1}{\tan a}$

• $\cos\left(\frac{\pi}{2} - a\right) = \sin a.$

Q2 (*) $\cos x \sin x + \frac{\sqrt{3}-1}{2}\cos^2 x \leq \frac{\sqrt{3}+1}{2}\sin^2 x$

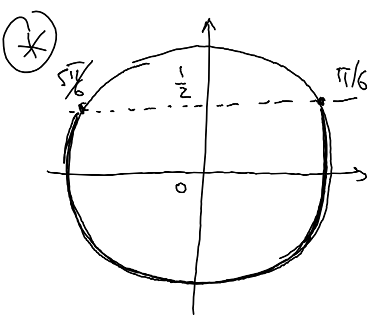
$\Leftrightarrow \cos x \sin x + \frac{\sqrt{3}}{2}\left(\cos^2 x - \sin^2 x\right) - \frac{1}{2}\left(\cos^2 x + \sin^2 x\right) \leq 0$

$\Leftrightarrow \frac{1}{2}\sin 2x + \frac{\sqrt{3}}{2}\cos 2x \leq \frac{1}{2}$

$\Leftrightarrow \cos\frac{\pi}{3}\sin 2x + \sin\frac{\pi}{3}\cos 2x \leq \frac{1}{2}$

$\Leftrightarrow \sin\left(2x + \frac{\pi}{3}\right) \leq \frac{1}{2}$

(*)



sur $[0; 2\pi[$:

$0 \leq 2x + \frac{\pi}{3} \leq \frac{\pi}{6}$ et $\frac{5\pi}{6} \leq 2x + \frac{\pi}{3} < 2\pi$

$\boxed{-\frac{\pi}{6} \leq x \leq -\frac{\pi}{12}}$ et $\boxed{+\frac{\pi}{2} \leq x < \frac{5\pi}{6}}$

K22

Qq rappels :

* $\tan(a+b) = \dfrac{\tan a + \tan b}{1 - \tan a \tan b}$

* $\tan(2a) = \dfrac{2\tan a}{1-\tan^2 a}$

* $\tan a = \tan b \iff a = b + k\pi,\ k \in \mathbb{Z}$.

Etape 1 : Mq $\underbrace{\tan\left(2\arctan\frac{1}{3} + \arctan\frac{1}{7}\right)}_{A} = 1$.

$$A = \frac{\tan(2\arctan 1/3) + \tan(\arctan 1/7)}{1 - \tan(2\arctan 1/3) \times \tan(\arctan 1/7)}$$

Or $\tan(2\arctan 1/3) = \dfrac{2\tan(\arctan 1/3)}{1-\tan^2(\arctan 1/3)} = \dfrac{2/3}{1-\frac{4}{9}} = \dfrac{2}{3}\times\dfrac{9}{8} = \dfrac{3}{4}$

et $\tan(\arctan 1/7) = 1/7$

D'où $A = \dfrac{\frac{3}{4}+1/7}{1-\frac{3}{4}\times\frac{1}{7}} = \dfrac{21+4}{28} \times \dfrac{28}{28-3} = 1$.

On déduit que $\boxed{\underbrace{2\arctan\frac{1}{3} + \arctan\frac{1}{7}}_{B} = \frac{\pi}{4} + k\pi,\ k\in\mathbb{Z}.}$

On sait aussi que pour $x \geq 0$ on a : $0 \leq \arctan x \leq x$

D'où $\frac{\pi}{4} - \pi < 0 \leq B \leq \frac{2}{3} + \frac{1}{7} \leq \frac{17}{21} < \frac{\pi}{4} + \pi$

D'où $B = \frac{\pi}{4}$